Curtin University, Semester 1, 2022
ECON 4002 (Dr. Lei Pan)
Problem Set 1
Due Friday, April 1st at 5:00pm AWST

Question 1. [30 marks] Solow-Swan model

Suppose the production function is Cobb-Douglas. (i.e. $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$)

(a) [10 marks] Find expressions for k^*, y^* and c^* as functions of the parameters of the models, s, n, δ, g and α.

(b) [10 marks] What is the golden-rule value of k?

(c) [5 marks] What saving rate is needed to yield the golden-rule capital stock?

(d) [5 marks] Describe how, if at all, each of the following developments affects the steady state capital per unit of effective labour in the Solow-Swan model (*Hint: Think about how lines $sf(k)$ and $(n+g+\delta)$ would shift*).

(i) [3 marks] The rate of technological progress rises.

(ii) [2 makrs] The saving rate rises.

Question 2. [20 marks] Consumer maximisation problem (2 periods)

Solve the following optimisation problem of the representative consumer:

$$\max_{c_1,c_2,k_2} V_1 = \max_{c_1,c_2,k_2} U(c_1) + \beta U(c_2)$$

subject to

$$c_1 + k_2 = F(k_1) + (1-\delta)k_1$$
$$c_2 = F(k_2) + (1-\delta)k_2$$

Period utility is given by:

$$U(c_t) = \log c_t$$

and output is given by:

$$y_t = F(k_t) = k_t^{\alpha}$$

where $t = 1, 2$.

(a) [15 marks] Derive the Euler equation for the specific utility- and production function by employing the Lagrange approach.

(b) [5 marks] What does your result say about the consumption path of the consumer?

Question 3. **[50 marks] Social security in the Diamond model**

Consider a Diamond economy where g is zero, production function is Cobb-Douglas, and utility is logarithmic.

Pay-as-you-go social security

Suppose the government taxes each young individual an amount τ and uses the proceeds to pay benefits to old individuals; thus each old person receives $(1+n)\tau$ (government has a balanced budget).

(a) [20 marks] Write down the individual's utility maximisation problem, derive the first order condition, solve for s_t and find $\frac{\partial s_t}{\partial \tau}$, $\frac{\partial c_{1t}}{\partial \tau}$ and $\frac{\partial c_{2t+1}}{\partial \tau}$. Discuss how s_t, c_{1t} and c_{2t+1} change with τ.

(b) [10 marks] Write k_{t+1} as a function of k_t. Is k^* bigger or smaller than the old value of k^* when the social security is absent, i.e. if $\tau = 0$ (*Hint: You do not need to solve for* k^*. *Just compare the transition equations in two cases to see whether introducing the social security shifts the transition equation up or down. Then you will find out whether* k^* *is bigger or smaller*).

Fully funded social security

Suppose the government taxes each young person an amount τ and uses the proceeds to purchase capital. Individuals born at t therefore receive $(1+r_{t+1})\tau$ when they are old.

(c) [10 marks] Write down the individual's utility maximisation problem, derive the first order condition and solve for s_t. Find $\frac{\partial s_t}{\partial \tau}$ and explain the effect of τ on private saving.

(d) [5 marks] Write k_{t+1} as a function of k_t. How does k^* differ from the old value of k^* when the social security is absent? And why?

(e) [5 marks] Can the government use fully funded social security to improve on a decentralised equilibrium?